

The Berry phase in inflationary cosmology

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We derive an analogue of Berry phase associated with inflationary cosmological perturbations by obtaining the corresponding wavefunction. We have further shown that cosmological Berry phase can be completely envisioned through the observable parameters. So one can, in principle, establish a supplementary probe of inflationary cosmology through the measurement of the associated Berry phase. Finally, we discuss some possible consequences of this quantity in inflationary cosmology.

I. INTRODUCTION

Since its inception Berry phase has drawn a lot of attention in physics community. Although the geometric phase was known long ago *a la* Aharonov-Bohm effect, the general context of a quantum-mechanical state developing adiabatically in time under a slowly varying parameter-dependent Hamiltonian has been analyzed by Berry [1] who argued that when the parameters return adiabatically to their initial values after traversing a closed path, the wavefunction acquires a *geometric phase factor* depending on the path, in addition to the well-known *dynamical phase factor*. The existence of a geometric phase in an adiabatic evolution is not only confined to quantum phenomenon, the classical analogue of it also exists and is referred to as the Hannay angle [2]. Berry established a semiclassical relation between the quantum and classical geometric phases in adiabatic evolution [3]. The Berry phase has been the subject of a variety of theoretical and experimental investigations [4]; possible applications range from quantum optics and molecular physics to fundamental quantum mechanics and quantum computation. Some analyses have been made to study this phase in the area of cosmology and gravitation also. In particular, the Berry phase has been calculated in the context of relic gravitons [5]. In [6] a covariant generalization of the Berry phase has been obtained. Investigations were also made to study the behavior of a scalar particle in a class of stationary spacetime backgrounds and the emergence of Berry phases in the dynamics of a particle in the presence of a rotating cosmic string [7]. The gravitational analogue of the Aharonov-Bohm effect in the spinning cosmic string spacetime background was also obtained [8]. Within a typical framework of cosmological model the Berry phase has been shown to be associated with the decay width of the state in case of some well known examples of vacuum instability [9].

The inflationary scenario [10] – so far the most physically motivated paradigm for early universe – is also in vogue for quite some time now. Among other motiva-

tions, the inflationary scenario is successful to a great extent in explaining the origin of cosmological perturbation seeds [11]. The accelerated expansion converts the initial vacuum quantum fluctuations into macroscopic cosmological perturbations. So, measurement of any quantum property which reflects on classical observables serve as a supplementary probe of inflationary cosmology, complementing the well-known CMB polarization measurements [12, 13]. Though this is an important issue, we notice that there has been very little study in the literature which deals with proposing measurable quantities which may measure the genuine quantum property of the seeds of classical cosmological perturbations. The only proposition that has drawn our attention is via violation of Bell's inequality [14]. This has led us to investigate for the potentiality of Berry phase in providing a measurable quantum property which is inherent in the macroscopic character of classical cosmological perturbations, thereby serving as a supplementary probe to CMB in inflationary cosmology.

In this article our primary intention is to demonstrate the effect of the curved spacetime background in the dynamical evolution of the quantum fluctuations during inflation through the derivation of the associated Berry phase and search for the possible consequences via observable parameters. The quantum fluctuations in inflation are realized by Mukhanov-Sasaki equation which is analogous to time dependent harmonic oscillator equation. The associated physical mechanism for cosmological perturbations can be reduced to the quantization of a parametric oscillator leading to particle creation due to interaction with the gravitational field and may be termed as cosmological Schwinger effect [15]. The relation between the Berry phase and the Hannay angle has been studied for the generalized time dependent harmonic oscillator [16]. This relation is also extended [17] from adiabatic to non-adiabatic time dependent harmonic oscillator. Stimulated by these, one may expect to derive the cosmological analogue of Berry phase in the context of inflationary perturbations and search for possible consequences via observable parameters. To this end, we first find an exact wavefunction for the system of inflationary cosmological perturbation by solving the associated *Schrodinger equation*. The relation [18, 19] between the dynamical invariant [20–22] and the geometric phase has then been utilized to derive the corresponding

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Berry phase. For slow roll inflation the total accumulated phase gained by each of the modes during sub-Hubble oscillations (adiabatic limit) is found to be a new parameter made of corresponding (scalar and tensor) spectral indices. So in principle, measurement of the Berry phase of the quantum cosmological perturbations provide us an indirect route to estimate spectral indices and other observable parameters therefrom. Further, since tensor spectral index is related to the tensor to scalar amplitude ratio through the consistency relation, the Berry phase can indeed be utilized to act as a supplementary probe of inflationary cosmology.

II. COSMOLOGICAL PERTURBATION AS A TIME DEPENDENT HARMONIC OSCILLATOR

Let us start with the usual gauge invariant [24] technique in which the action for perturbations of scalar modes can be equivalently expressed upto a total time derivative term as [25]

$$\mathbf{S}^S = \frac{1}{2} \int d\eta d\mathbf{x} \left[\Pi^2 + \frac{z'}{z} (\Pi v - v \Pi) - \delta^{ij} \partial_i v \partial_j v \right] \quad (2.1)$$

where $\Pi \equiv \frac{\partial \mathbf{L}}{\partial v'} = v' - \frac{z'}{z} v$. Here v is related to the comoving curvature perturbation \mathcal{R} via the relation $v = -z\mathcal{R}$ where $z \equiv \frac{a\phi}{\mathcal{H}(\eta)}$ and $\mathcal{H} \equiv \frac{a'}{a}$ being the conformal Hubble parameter and ϕ is the scalar field driving the inflation.

Promoting the fields to operators and taking the Fourier decomposition, the Hamiltonian operator corresponding to the above action (2.1) is found to be

$$\begin{aligned} \hat{\mathbf{H}}_{\mathbf{k}}^S &= \frac{1}{2} \left[\hat{\Pi}_{1\mathbf{k}}^2 + \frac{z'}{z} \left(\hat{\Pi}_{1\mathbf{k}} \hat{v}_{1\mathbf{k}} + \hat{v}_{1\mathbf{k}} \hat{\Pi}_{1\mathbf{k}} \right) + k^2 \hat{v}_{1\mathbf{k}}^2 \right] \\ &+ \frac{1}{2} \left[\hat{\Pi}_{2\mathbf{k}}^2 + \frac{z'}{z} \left(\hat{\Pi}_{2\mathbf{k}} \hat{v}_{2\mathbf{k}} + \hat{v}_{2\mathbf{k}} \hat{\Pi}_{2\mathbf{k}} \right) + k^2 \hat{v}_{2\mathbf{k}}^2 \right] \\ &\equiv \hat{\mathbf{H}}_{1\mathbf{k}}^S + \hat{\mathbf{H}}_{2\mathbf{k}}^S \end{aligned} \quad (2.2)$$

where we have decomposed $v_{\mathbf{k}} \equiv v_{1\mathbf{k}} + i v_{2\mathbf{k}}$ into its real and imaginary part.

Similarly, the action for tensor perturbations is

$$\mathbf{S}^T = \frac{M_P^2}{2} \int d\eta d\mathbf{x} \frac{a^2}{2} \left[h'^2 - \delta^{ij} \partial_i h \partial_j h \right] \quad (2.3)$$

By means of the substitution $u = \frac{M_P}{\sqrt{2}} h a$ and taking the Fourier decomposition, the Hamiltonian operator corresponding to the tensor perturbations is found to be

$$\begin{aligned} \hat{\mathbf{H}}_{\mathbf{k}}^T &= \frac{1}{2} \left[\hat{\pi}_{1\mathbf{k}}^2 + \frac{a'}{a} \left(\hat{\pi}_{1\mathbf{k}} \hat{u}_{1\mathbf{k}} + \hat{u}_{1\mathbf{k}} \hat{\pi}_{1\mathbf{k}} \right) + k^2 \hat{u}_{1\mathbf{k}}^2 \right] \\ &+ \frac{1}{2} \left[\hat{\pi}_{2\mathbf{k}}^2 + \frac{a'}{a} \left(\hat{\pi}_{2\mathbf{k}} \hat{u}_{2\mathbf{k}} + \hat{u}_{2\mathbf{k}} \hat{\pi}_{2\mathbf{k}} \right) + k^2 \hat{u}_{2\mathbf{k}}^2 \right] \\ &\equiv \hat{\mathbf{H}}_{1\mathbf{k}}^T + \hat{\mathbf{H}}_{2\mathbf{k}}^T \end{aligned} \quad (2.4)$$

where we have decomposed $u_{\mathbf{k}} \equiv u_{1\mathbf{k}} + i u_{2\mathbf{k}}$ similarly.

Thus, for both the scalar and tensor modes, the Hamiltonian is a sum of two generalized time dependent harmonic oscillators, each of them having the form

$$\hat{\mathbf{H}}_{j\mathbf{k}} = \frac{1}{2} \left[k^2 \hat{q}_{j\mathbf{k}}^2 + Y(\eta) (\hat{p}_{j\mathbf{k}} \hat{q}_{j\mathbf{k}} + \hat{q}_{j\mathbf{k}} \hat{p}_{j\mathbf{k}}) + \hat{p}_{j\mathbf{k}}^2 \right] \quad (2.5)$$

where $\hat{q}_{j\mathbf{k}} = \hat{v}_{j\mathbf{k}}, \hat{u}_{j\mathbf{k}}; \hat{p}_{j\mathbf{k}} = \hat{\Pi}_{j\mathbf{k}}, \hat{\pi}_{j\mathbf{k}}$ and $Y = \frac{z'}{z}, \frac{a'}{a}$ for the scalar and tensor modes respectively and $j = 1, 2$ with the frequency given by $\omega = \sqrt{k^2 - Y^2}$.

III. BERRY PHASE THROUGH DYNAMICAL INVARIANT OPERATOR METHOD

It has become evident that the Lewis-Risenfeld invariant formulation [20] can be applied to the treatment of time dependent quantum system if a invariant can be found. The invariant formulation for obtaining the exact solution for the systems with time dependent Hamiltonian is closely related to the study of the phases.

Here, the situation may be analyzed by solving the associated Schrodinger equation

$$\hat{\mathbf{H}}_{\mathbf{k}} \Psi = (\hat{\mathbf{H}}_{1\mathbf{k}} + \hat{\mathbf{H}}_{2\mathbf{k}}) \Psi = i \frac{\partial}{\partial \eta} \Psi \quad (3.1)$$

We shall now make use of the *dynamical invariant operator method* [20] to obtain the Berry phase for the present system. In this method, we first look for a non-trivial hermitian operator $I_{\mathbf{k}}(\eta)$ satisfying the Liouville-von Neumann equation

$$\frac{dI_{\mathbf{k}}}{d\eta} = -i [I_{\mathbf{k}}, \mathbf{H}_{\mathbf{k}}] + \frac{\partial I_{\mathbf{k}}}{\partial \eta} = 0. \quad (3.2)$$

Whenever such an invariant operator exists provided it does not contain time derivative operator, one can write down the solutions of the Schrodinger equation in the form

$$\Psi_n = e^{i\alpha_n(\eta)} \Theta_n, \quad n = 0, 1, 2, \dots \quad (3.3)$$

where Θ_n are the eigenfunctions of the operator $I_{\mathbf{k}}$ and $\alpha_n(\eta)$ are known as the Lewis phase. Here, $\mathbf{H}_{\mathbf{k}} = \mathbf{H}_{1\mathbf{k}} + \mathbf{H}_{2\mathbf{k}}$ and the invariant operator associated to this Hamiltonian can be expressed as

$$I_{\mathbf{k}}(\eta) \equiv I_1(q_{1\mathbf{k}}, \eta) + I_2(q_{2\mathbf{k}}, \eta) \quad (3.4)$$

Following the usual technique [20–22, 26] we obtain

$$\begin{aligned} I_{\mathbf{k}} &= \frac{1}{2} \left[\frac{q_{1\mathbf{k}}^2}{\rho_k^2} + \left(\rho_k \left[p_{1\mathbf{k}} + Y q_{1\mathbf{k}} \right] - \rho'_k q_{1\mathbf{k}} \right)^2 \right] \\ &+ \frac{1}{2} \left[\frac{q_{2\mathbf{k}}^2}{\rho_k^2} + \left(\rho_k \left[p_{2\mathbf{k}} + Y q_{2\mathbf{k}} \right] - \rho'_k q_{2\mathbf{k}} \right)^2 \right] \\ &= I_1 + I_2 \end{aligned} \quad (3.5)$$

where ρ_k is a time dependent real function satisfying the Milne-Pinney equation

$$\rho_k'' + \Omega^2(\eta, k)\rho_k = \frac{1}{\rho_k^3(\eta)} \quad (3.6)$$

with $\Omega^2 = \omega^2 - \frac{dY}{d\eta}$. To find the solutions of the Schrodinger Eqn.(3.1) we need the eigenstates of the operator I_k governed by the eigenvalue equation

$$I_k \Theta_{n_1, n_2}(q_{1k}, q_{2k}, \eta) = \lambda_{n_1, n_2} \Theta_{n_1, n_2}(q_{1k}, q_{2k}, \eta) \quad (3.7)$$

The eigenstates of the operator I_k turn out to be

$$\begin{aligned} \Theta_{n_1, n_2} &= \frac{\bar{H}_{n_1} \left[\frac{q_{1k}}{\rho_k} \right] \bar{H}_{n_2} \left[\frac{q_{2k}}{\rho_k} \right]}{\sqrt[4]{\pi^2 2^{2(n_1+n_2)} (n_1! n_2!)^2 \rho_k^4}} \times \\ &\exp \left[\frac{i}{2} \left(\frac{\rho_k'}{\rho_k} - Y(\eta) + \frac{i}{\rho_k^2} \right) (q_{1k}^2 + q_{2k}^2) \right] \end{aligned} \quad (3.8)$$

where \bar{H}_n are the Hermite polynomials of order n and the associated eigenvalues are given by

$$\lambda_{n_1, n_2} = \left(n_1 + \frac{1}{2} \right) + \left(n_2 + \frac{1}{2} \right) \quad (3.9)$$

The Lewis phases can be found from its definition

$$\frac{d\alpha_{n_1, n_2}}{d\eta} = \left\langle \Theta_{n_1, n_2} \left| i \frac{\partial}{\partial \eta} - \hat{H}_k \right| \Theta_{n_1, n_2} \right\rangle \quad (3.10)$$

resulting in

$$\alpha_{n_1, n_2} = -(n_1 + n_2 + 1) \int \frac{d\eta}{\rho_k^2} \quad (3.11)$$

As a consequence the eigenstates of the Hamiltonian are now completely known and are given by

$$\begin{aligned} \Psi_{n_1, n_2} &= \frac{e^{i\alpha_{n_1, n_2}(\eta)} \bar{H}_{n_1} \left[\frac{q_{1k}}{\rho_k} \right] \bar{H}_{n_2} \left[\frac{q_{2k}}{\rho_k} \right]}{\sqrt[4]{\pi^2 2^{2(n_1+n_2)} (n_1! n_2!)^2 \rho_k^4}} \times \\ &\exp \left[\frac{i}{2} \left(\frac{\rho_k'}{\rho_k} - Y(\eta) + \frac{i}{\rho_k^2} \right) (q_{1k}^2 + q_{2k}^2) \right] \end{aligned} \quad (3.12)$$

The eigenstates of the Hamiltonian for cosmological perturbations can be found exactly provided it possesses a dynamical invariant containing no time derivative operation.

Once the Lewis phase is calculated, this can be utilized in deriving the Berry phase associated with the system corresponding to the particle creation through the vacuum quantum fluctuations during inflation. Using Eqns.(3.10) and (3.11) we obtain the corresponding

Berry phase

$$\begin{aligned} \gamma_{n_1, n_2, k} &\equiv i \int_0^\Gamma \left\langle \Theta_{n_1, n_2} \left| \frac{\partial}{\partial \eta} \right| \Theta_{n_1, n_2} \right\rangle d\eta \\ &= -\frac{1}{2} (n_1 + n_2 + 1) \int_0^\Gamma \left(\frac{1}{\rho_k^2} - \rho_k^2 \omega^2 - (\rho_k')^2 \right) d\eta \end{aligned} \quad (3.13)$$

where it has been assumed that the invariant $I_k(\eta)$ is Γ periodic and its eigenvalues are non-degenerate. To get a deeper physical insight the quantitative estimation of the Berry phase is very important. Eqn.(3.13) tells us that for this estimation, the knowledge of ρ_k is essential but the solution of Eqn.(3.6) is difficult to obtain. Another point to be carefully handled is to set the value of the parameter Γ . Keeping all these in mind and considering compatible physical conditions we proceed as follows.

First we note that in the adiabatic limit (which is quite justified for sub-Hubble modes) Eqn.(3.6) can be solved [22] by a series of powers in adiabatic parameter, δ (< 1) as

$$\rho_k = \rho_0 + \delta \rho_1 + \delta^2 \rho_2 + \dots \quad (3.14)$$

with zeroth order term given by $\rho_0 = \omega^{-\frac{1}{2}}$. Thus for the ground state of the system, in the adiabatic limit, the Berry phase for a particular cosmological perturbation mode can be evaluated upto the first order in δ , which is given by

$$\gamma_k^{(S, T)} = \frac{1}{2} \int_0^\Gamma \frac{Y'}{\sqrt{k^2 - Y^2}} d\eta \quad (3.15)$$

One may note that our result (3.15) coincides with that of Berry [3].

Our next task is to fix the value of the parameter Γ . To this end we shall calculate the total Berry phase accumulated by each mode during inflationary evolution. For the ground state of the system this turns out to be

$$\gamma_k^{S, T}{}_{total} = -\frac{1}{2} \lim_{\eta' \rightarrow -\infty} \int_{\eta'}^0 \left(\frac{1}{\rho_k^2} - \rho_k^2 \omega^2 - (\rho_k')^2 \right) d\eta \quad (3.16)$$

where the superscripts S and T stand for scalar and tensor modes respectively. A non-zero value of the parameter $\gamma_k^{S, T}{}_{total}$ will ensure that there are some nontrivial effects of the curved space-time background on the evolution of the quantum fluctuations and may play an important role in the growth of inflationary cosmological perturbations. It worths mentioning that if one calculates the *accumulated Berry phase* using Eqn.(3.16) for the *super-Hubble* modes satisfying $k^2 < [Y(\eta)]^2$, the frequency of the system becomes *imaginary*. But so far as one is concerned about evolution of sub-Hubble modes, the above analysis is a very good approximation to the actual physical scenario. In the present article this is what we are interested in.

IV. PHYSICAL IMPLICATION OF BERRY PHASE IN COSMOLOGICAL PARAMETERS

Let us now set up the link between this parameter and the cosmological observables. In the adiabatic limit, *accumulated Berry phase* during sub-Hubble oscillations of the each mode is given by

$$\gamma_{k\ sub}^{S,T} = -\frac{1}{2} \lim_{\eta' \rightarrow -\infty} \int_{\eta'}^{\eta_0^{S,T}} \frac{Y'}{\sqrt{k^2 - Y^2}} d\eta \quad (4.1)$$

where $\eta_0^{S,T}$ is the conformal time which satisfies the relation $k^2 = [Y(\eta_0^{S,T})]^2$ and guarantees that the modes are within the horizon and oscillating with real frequency. The formula (4.1) is adopted to derive the relations between the accumulated Berry phase and the cosmological observable parameters. From now on we shall drop the subscript ‘sub’ keeping in mind that the calculations are for sub-horizon modes only.

The *accumulated Berry phase* during sub-Hubble evolution of the scalar modes, in terms of the slow-roll parameters, turns out to be

$$\begin{aligned} \gamma_k^S &= \frac{1}{2} \lim_{\eta' \rightarrow -\infty} \int_{\eta'}^{-\frac{\sqrt{1+6\epsilon_1-2\epsilon_2}}{k}} \frac{\frac{z''}{z} - \left(\frac{z'}{z}\right)^2}{\sqrt{k^2 - \left(\frac{z'}{z}\right)^2}} d\eta \\ &\approx -\frac{\pi}{4} \frac{1+3\epsilon_1-\epsilon_2}{\sqrt{1+6\epsilon_1-2\epsilon_2}} \end{aligned} \quad (4.2)$$

For brevity, we have restricted our analysis upto the first order in *slow-roll* parameters and we have neglected time variation in ϵ_1, ϵ_2 . And for the tensor modes we have

$$\begin{aligned} \gamma_k^T &= \frac{1}{2} \lim_{\eta' \rightarrow -\infty} \int_{\eta'}^{-\frac{\sqrt{1+2\epsilon_1}}{k}} \frac{\frac{a''}{a} - \left(\frac{a'}{a}\right)^2}{\sqrt{k^2 - \left(\frac{a'}{a}\right)^2}} d\eta \\ &\approx -\frac{\pi}{4} \frac{1+\epsilon_1}{\sqrt{1+2\epsilon_1}} \end{aligned} \quad (4.3)$$

In the above derivations we have made use of the standard definition of the slow-roll parameters [27]

$$\epsilon_1 \equiv \frac{M_P^2}{2} \left(\frac{V'(\phi)}{V(\phi)} \right)^2, \quad \epsilon_2 \equiv M_P^2 \left(\frac{V''(\phi)}{V(\phi)} \right), \quad (4.4)$$

$V(\phi)$ being inflaton potential. For the estimation of $\gamma_k^{S,T}$, the slow-roll parameters are to be evaluated at the start of inflation. But during inflation the slow-roll parameters does not evolve significantly from their initial values for first few e -folds, which is relevant for the present day observable modes as they are supposed to leave the horizon during first 10 e -folds. So in the above estimates for $\gamma_k^{S,T}$ we can consider ϵ_1 and ϵ_2 as their values at horizon crossing without committing any substantial error.

We are now in a position to relate this phase with observable parameters. At the horizon exit the fundamental observable parameters can be expressed in terms of slow-roll parameters (upto the first order in ϵ_1, ϵ_2) as [27, 28]

$$\begin{aligned} P_{\mathcal{R}} &= \frac{V}{24\pi^2 M_P^4 \epsilon_1}, \quad n_S = 1 + 2\epsilon_2 - 6\epsilon_1 \\ n_T &= -2\epsilon_1, \quad r = 16\epsilon_1 \end{aligned} \quad (4.5)$$

where $P_{\mathcal{R}}$ is the scalar power spectrum, n_S and n_T are the scalar and tensor spectral indices respectively, r is the tensor to scalar ratio. As a consequence the *accumulated Berry phase* associated with the *sub-Hubble* oscillations of the scalar fluctuations during inflation can be expressed in terms of the observable parameters (and vice versa) as follows

$$\gamma_k^S \approx -\frac{\pi}{8} \frac{3 - n_S(k)}{\sqrt{2 - n_S(k)}} \quad (4.6)$$

$$n_S(k) \approx 3 - \frac{8\gamma_k^S}{\pi} \left(\frac{4\gamma_k^S}{\pi} - \sqrt{\frac{16[\gamma_k^S]^2}{\pi^2} - 1} \right) \quad (4.7)$$

Therefore accumulated Berry phase for the scalar modes is related to the scalar spectral index. For the tensor modes the corresponding expressions turn out to be

$$\gamma_k^T \approx -\frac{\pi}{8} \frac{2 - n_T(k)}{\sqrt{1 - n_T(k)}} \quad (4.8)$$

$$n_T(k) \approx 2 - \frac{8\gamma_k^T}{\pi} \left(\frac{4\gamma_k^T}{\pi} - \sqrt{\frac{16[\gamma_k^T]^2}{\pi^2} - 1} \right) \quad (4.9)$$

Eqns.(4.6) and (4.8) reveal that the Berry phase due to scalar and tensor modes basically correspond to a new parameter made of corresponding spectral indices.

Further, the accumulated Berry phase associated with the total gravitational fluctuations (a sumtotal of γ_k^S and γ_k^T) can be expressed in terms of the other observable parameter as well, giving

$$\gamma_k \equiv \gamma_k^S + \gamma_k^T \approx -\frac{\pi}{8} \left[\frac{3 - n_S(k)}{\sqrt{2 - n_S(k)}} + \frac{2 + \frac{r}{8}}{\sqrt{1 + \frac{r}{8}}} \right] \quad (4.10)$$

$$\approx -\frac{\pi}{8} \left[\frac{3 - n_S(k)}{\sqrt{2 - n_S(k)}} + \frac{2 + \frac{V}{12\pi^2 M_P^4 P_{\mathcal{R}}}}{\sqrt{1 + \frac{V}{12\pi^2 M_P^4 P_{\mathcal{R}}}}} \right] \quad (4.11)$$

Therefore the *accumulated Berry phase* for sub-Hubble oscillations of the perturbation modes during inflation can be completely envisioned through the observable parameters. The estimation of the Berry phase gives a deeper physical insight of the quantum property of the inflationary perturbation modes. As a result, at least in principle, we can claim that measurement of Berry phase can serve as a probe of quantum properties reflected on classical observables.

The physical implication of Berry phase in cosmological parameters transpires out from the above analysis. In a nutshell, the classical cosmological perturbation

modes (both scalar and tensor) having quantum origin picks up a phase during their advancement through the curved space-time background that depends entirely on the background geometry and can be estimated quantitatively by measuring the corresponding spectral indices. So the Berry phase for the quantum counterpart of the classical cosmological perturbations endow us with the measure of spectral index.

V. DISCUSSION AND FUTURE PROSPECTS

The current observations from WMAP7 [12] have put stringent constraints on n_S ($0.948 < n_S < 1$) but only an upper bound for r has been reported so far ($r < 0.36$ at 95% C.L.), with PLANCK [13] expecting to survey upto the order of 10^{-2} . Given this status, any attempt towards the measurement of cosmological Berry phase may thus reflect observational credentials of this parameter in inflationary cosmology. For example, it is now well-known that any conclusive comment on the energy scale of inflation (V in Eqn. (4.10)) provides crucial information about fundamental physics. However, in CMB polarization experiments, the energy scale cannot be conclusively determined because there is a degeneracy between E and B modes via the first slow roll parameter ϵ_1 (Eqn. 4.5), which can only be sorted out once r is measured conclusively. But B mode polarized states can be contaminated with cosmic strings, primordial magnetic field etc, thereby making it difficult to measure r conclusively (for a lucid discussion see [29]). So, cosmological Berry phase may have the potentiality to play some important role in inflationary cosmology, since it is related to r and V via Eqns. (4.10) and (4.11).

So far as the detection of cosmological Berry phase is concerned, we are far away from quantitative measurements. A possible theoretical aspect of detection [30] of the analogue of cosmological Berry phase may be developed in squeezed state formalism [25]. For a quantum harmonic oscillator, when a squeezing Hamiltonian is switched on, and the squeezing parameter is varied, we can find a detectable Berry phase. As the inflationary perturbations can be studied in the squeezed state formalism, we hope to put forward our analysis on the

detection of the geometric phase in near future.

On principle Berry phase can be measured from an experiment dealing with phase difference (e.g. interference). Recently, an analogy between phonons in an axially time-dependent ion trap and quantum fields in an expanding/contracting universe has been derived and corresponding detection scheme for the analogue of cosmological particle creation has been proposed which is feasible with present-day technology [31]. Besides, there exists [25] a scheme for measuring the Berry phase in the vibrational degree of freedom of a trapped ion. We hope that these type of detection schemes may be helpful for observation of the cosmological analogue of the Berry phase in laboratory in future.

VI. SUMMARY

In this article we have demonstrated how the exact expression for the wave function of the quantum cosmological perturbations can be analytically obtained by solving the associated Schrodinger equation following the dynamical invariant technique. This helps us to derive an expression for cosmological analogue of Berry phase.

We have also demonstrated how this quantity is related to cosmological parameters and how these relations can be utilized in extracting further information related to the observational aspects of inflationary perturbations. We end up with this optimistic note that any measurements of this quantum property may thus reflect on inflationary cosmology as a supplementary probe in measuring classical observables.

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